## Designing plans for Khufu's Pyramid using grids

A gigantic construction site such as that of Khufu's Pyramid is certainly not the result of a project conceived on a day-to-day basis, but on the contrary of a conception and programming decided far in advance that left nothing to chance. In terms of design, the Egyptians came up with an elegant and highly efficient solution: the use of grids based on the Egyptian cubit.

First and foremost, the Egyptian units were the cubit (royal) divided into 7 palms, each palm being itself divided into 4 fingers; there were therefore 28 fingers in a cubit.


In relation to our metric system, this gives:
1 cubit $=0.5236 \mathrm{~m}$ or 52.36 cm
1 palm $=0.0748 \mathrm{~m}$ or 7.48 cm
1 finger $=0.0187 \mathrm{~m}$ or 1.87 cm
It's already clear that the Egyptians were remarkably precise in their dimensions and measurements. With such a system, they could make any construction they wanted, their margin of error being very small, on the order of half a finger.


An Egyptian one cubit ruler (musée du Louvre)

Right now, everyone's mistake is to work on the pyramids using our metric system; we're making mistakes because we're not thinking in terms of "Ancient Egypt" at all...

Secondly, as the Egyptians had no 3D software at their disposal, they still had to imagine in space. A pyramid is first and foremost a volume, a 3-dimensional object that can be compared to an enclosed space in which rooms are to be built. It was therefore necessary to materialize this space in at least 2 dimensions: the Egyptians therefore worked on 2 planes, a horizontal and a vertical plane (also called the frontal plane). And to locate each component in space, the Egyptians used a system of square orthogonal grids based on the unit, a cubit, or a multiple of this unit as required.

Thus, the pyramid's volume itself is designed on two primary grids determined in multiples of 20 cubits:

- on the horizontal plane, a basic grid of 22 times 20 cubits, i.e., 440 cubits
- on the vertical plane, a basic grid of 14 times 20 cubits, i.e., 280 cubits

This gives a Seked of 14/11 (14 being the height and 11 being the half-base at the apothem, i.e., $22 / 2=11$ ).

For the principle to work perfectly, i.e., the transmission of what has been conceived into constructed reality, an unchanging point of reference is needed: this is the vertical axis materialized by the junction of the North/South and East/West axes.
The primary grid on the horizontal plane is therefore centered on the North/South and East/West axes, and this is why for construction, apart from the religious aspect of this orientation, determining North is fundamental to design, as it is the simplest and most precise method of determining a basic axis on the building site. The referencing system thus becomes, for the volume, a few easy-to-transcribe lines for construction purposes.
Once the North/South axis has been determined by astronomy and confirmed by the sun's shadow, the East/West axis is easily determined by successive arcs of circles. The 4 bases of the faces are traced:

Base of East face 220 cubits (22 times 20c) east of North/South axis
Base of West face 220 cubits (22 times 20c) west of North/South axis
Base of North face 220 cubits ( 22 times 20c) north of East/West axis
Base of South face 220 cubits south of East/West axis

By starting from the axes instead of a face, the margin of error is already divided by 2. As for the angles, they are determined by the sections of the base lines and can be checked and specified from the diagonals.


The 20c $\times 20$ chorizontal primary grid is centered on the N/S and E/W axes marked in yellow.


The 20c x 20c vertical primary grid is centered on the vertical axis marked in yellow.


The combination of the 2 planes gives the volume.

As for the internal structures, they are designed and transcribed from a secondary grid of one square cubit drawn within the primary grid, but only in the area where the structures are to be built: a strip centered on the North/South axis and approximately 40 cubits wide, with the structures offset to the East within this strip.

All the structures built in Khufu's pyramid fall within this band. The precision of the structures (in relation to the North/South axis in particular) is due to the fact that on site, all these structures are in the immediate vicinity of this North/South axis, so the margin of error in relation to this axis is very small; in fact, it can be seen on site that it is in this zone, the Grand Gallery, the ascending, descending and horizontal corridors, that alignment with the North/South axis is the most precise, and it's easy to see why.

It's important to understand that before building the pyramid, the architects and engineers had to design it and draw up plans. To build a King's Chamber +43 m high 14 years after the start of construction at a precise location, everything had to be determined in advance.
What is the King's Chamber? Ultimately, it's a shoebox-shaped volume (!) 20 cubits long, 10 cubits wide and 11 cubits high that needs to be positioned in space (the volume of the pyramid). Thanks to
the grid system, it is perfectly adjustable spatially. Because of the decision to give this chamber a flat ceiling, the Egyptians were obliged to integrate the Grand Gallery (a construction crane or, rather, a tractor) into the project. And it is this tractor that will determine the spatial layout of the volume.

Around the 4th/5th year of the project, construction of the ascending corridor had to begin. In the horizontal plane, it was therefore necessary to know exactly where on the layer under construction the workers were going to start this corridor, i.e., 120 cubits north of the East/West axis, its axis being offset 13 cubits to the East in relation to the North/South axis. The next step was simple: the slopes of this ascending corridor and the Grand Gallery are 1:2 (i.e., $50 \%$ ). They rise by one cubit every 2 cubits!

In the vertical (frontal) plane, everything is also very simple, since it's all based on a grid of one square cubit. We find the King's Chamber 82 cubits from the base, its North wall 13 cubits south of the East/West axis (the chamber being off-center), its West wall 6 cubits west of the North/South axis and its East wall 14 cubits from the North/South axis (this positioning is determined by the position of the Grand Gallery).

The Grand Gallery measures 82 cubits on horizontal projection (horizontal plane) and has a vertical drop of 41 cubits (1:2). Its South (high) wall is offset by 3 cubits to the South in relation to the East/West axis; this offset is intended to ensure that the face of the "high platform" is exactly on the East/West axis; its axis is offset by 13 cubits to the East in relation to the North/South axis.

The zero point for the entire design is located at this precise point on the upper platform of the Grand Gallery (shown in red on the vertical primary grid above), with all internal structures having been positioned and traced "back down" towards the base to determine the starting point for the ascending corridor to position all the chambers exactly.

Everything has been designed according to this principle and is fully verifiable in 3D design.


The 1c $\times 1 \mathrm{c}$ horizontal secondary grid is centered on the $\mathrm{N} / \mathrm{S}$ axis.


Zoom in on the off-center internal structures on the $1 \times 1 \mathrm{c}$ horizontal secondary grid.


The secondary 1c $\times 1 \mathrm{c}$ vertical grid
The Queen's Chamber is centered on the East/West axis.
The vertical face of the Grand Gallery's upper platform is aligned on the East/West axis. These 2 details are not due to chance, but for very specific reasons.

Without computers, without 3D, the Egyptians knew how to design in space very well and very simply: everything is easy to transcribe from conception to realization and EVERYTHING IS IN FULL CUBITS, no commas. Why put commas in a volume this size?

In addition to grid design, the Egyptians also made models of sensitive points, such as corridor junctions. That's why they made a model (dug into the ground) some fifty meters to the east of the pyramid. And this model is still there, giving us some very good indications of how they worked.

Here's a glimpse of what was fundamental to Egyptian thinking at the time when it came to designing monuments: simplicity and rationality.

Jean-Pierre Houdin
Based on the original study of September 2007
3D drawings created with CATIA software from Dassault Systèmes

